University of Connecticut *Spring 2025*

STAT 6894, Probability Theory II *Section 001*

- **Instructor** Vladimir Pozdnyakov
  Office AUST 324
  Email `Vladimir.Pozdnyakov@uconn.edu`
  Office Hours Tue/Thu 11AM-noon, AUST 324
- **Lectures** Tue/Thu 9:30-10:45, BCH 443
- **Text** A Probability Path
  Sidney I. Resnick
- **Syllabus**
  - **Expectation**: Product Spaces and Fubini's Theorem; Variance, Covariance and Covariance Matrix.
  - **Characteristic Functions**: Characteristic Functions: Definition and Basic Properties; Characteristic Function of Normal Distribution; Bochner's Theorem, Polya's Theorem, and Characteristic Functions of Lattice Distributions; Inversion Formula; Uniqueness Theorem, Inversion Formula for Z-valued RVs, Inversion Formula for Integrable CFs; Moments of Distributions and Smoothness of CFs; Smoothness of CFs and Moments of Distributions; Characteristic Function of Random Vectors; Multivariate Normal Distribution.
  - **Convergence Concepts and Weak Convergence**: Different Types of Convergence, the Iff Condition for a.s. Convergence; Convergence Graph Theorem; Uniform Convergence to a Continuous cdf; Weak Convergence of Probability Measures, Quantile Function Lemma; Skorohod's Theorem; Mapping Theorem, Uniform Integrability and Convergence in Distribution Proposition; Portmanteau Theorem; Slutsky's Theorem; Helly's selection Theorem; Relatively Compact Family of Probability Measures, Prokhorov's theorem; Continuity Theorem; Week Laws of Large Numbers, Poisson's Theorem, Central Limit Theorem; Lindeberg-Feller Theorem, Lyapunov Theorem; Cauchy Criterion for a.s. Convergence; Kolmogorov's Inequality; Two-Series and Three-Series Theorems; Toeplitz Lemma, Kronecker's Lemma; Strong LLN for I.I.D. Random Variables.
  - **Conditional Expectation and Martingales**: Conditional Expectation, Existence and Uniqueness of Conditional Expectation; Properties of Conditional Expectation: Linearity, Positivity, Monotonicity, Conditional Monotone Convergence Theorem, Conditional Fatou's Lemma, Conditional Dominated Convergence Theorem, Conditional Jensen's Inequality, Tower Property, Non-Anticipating Multiplier Property, Independence Property, and Geometrical Interpretation; Martingale: Definition, Examples of Martingales;

Gambling Theorem, Stopped Martingale Theorem, Doob's Optional Stopping Theorem; Doob's Upcrossing Lemma, Doob's Convergence Theorem; L_2-Bounded Martingale Convergence, Doob's Decomposition; UI Martingale Convergence, Levy's Convergence Theorem; Doob's Submartingale Inequality, Doob's L_p Maximal Inequality.

- **Exam**
  - Midterm exam — March 25, 9:30AM-10:45AM
  - Final exam — TBA
- **Grades**
  - grades are based on the following weighed sum: midterm (40%) + final exam (60%)
  - the final exam is cumulative
  - there will be no make-up exams